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ABSTRACT

This report, part of Rand's Labor and Population Studies Program, delves into sources of black/white income differentials. This report has as its purpose, the use of employment quotas as an analytical device for devising a priori notions of what the effects of government attempts to reduce employment discrimination might be. Following a formal solution to both a one-sector and a two-sector model of quotas, wherein the two-sector case quotas are imposed on only one sector, a number of simulations are presented. The simulations are an arithmetical exercise conducted to give order of magnitude estimates of the social cost of quota-induced income redistribution. The main conclusion is that without skill bumping quotas are expensive means of redistributing income. With skill bumping, quotas have the potential of redistributing income at costs that appear trivial, but there is a problem. Skill bumping presumes that workers are upgraded into better paying jobs than they would otherwise hold. Protagonists of affirmative action have argued that quotas for skilled minority workers create incentives for increasing minority skills and therefore provide longer run incentives for eliminating the skill component of majority/minority income differentials. That these effects are in fact ambiguous is established by examination of a very simple one-sector model in which quotas only redistribute income without social costs. In sum, the analysis presented suggests that the antagonists' fears of quota programs with high social costs alongside widespread sacrifices of employment standards may be ill-founded. On the contrary, it is the flexibility of standards that can dampen costs. As in many other cases, effects of quotas seem to be largely an empirical question. (Author/AM)

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# EMPLOYMENT QUOTAS FOR MINORITIES

PREPARED FOR THE DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE AND FOR THE DEPARTMENT OF LABOR

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U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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## PREFACE

This is one of a series of reports from Rand's Labor and Population Studies Program that delve into sources of black/white income differentials. The research was performed in part under a grant from the Office of the Assistant Secretary for Planning and Evaluation of the Department of Health, Education, and Welfare and in part under contract with the Office of the Assistant Secretary for Planning, Evaluation, and Research of the Department of Labor.

The present report is conjectural. Its purpose is to use employment quotas--affirmative action in the extreme--as an analytical device for deriving a priori notions of what the effects of government attempts to reduce employment discrimination might be. That is, this report is seen as preliminary to empirical work to measure these effects. In fact, the empirical work has run ahead of the theory. Indirect evidence of the effects of government on black/white male wage ratios for 1970 are presented in James P. Smith and Finis R. Welch, *Black/White Male Earnings and Employment: 1960-1970*, The Rand Corporation, R-1666-DOL, June 1975.

The author is indebted to Giora Hanoch, George Johnson, Gregg Lewis, John Riley, Daniel Saks, James P. Smith, Robert Willis, and Minnie Moore for insightful comments on numerous parts of the report and to Iva Maclennan for assistance in the simulations.

## SUMMARY

The most important feature of employment quotas is that, when rigorously enforced, they create economic shortages of workers designated by quotas--described here as skilled minority workers--that otherwise would not exist. As such, enforced quotas generate wage premiums for those the quota forces to be in short supply, and concurrent with these premiums, quotas create rationing problems. This report explores only one avenue for resolving the rationing problem in depth, but sketches qualitative implications for other forms of resolution. The solution considered consists of skill bumping or promoting unskilled minority workers to skilled job classes. This particular resolution comes close to what antagonists of affirmative action have described as incentives for "sacrificing standards." The most important conclusion of the analysis is that quota effects without skill bumping--when quotas are rigorously enforced and rationing problems are not resolved--can differ radically from effects of quotas with skill bumping.

If quotas have any effect other than simply changing the sectoral distribution of employment, the effect is to redistribute income from majority to (quota-designated) minority workers. But, to the extent that quotas change the sectoral distribution of production and the skill composition of production within sectors, aggregate production is less efficiently organized than it otherwise would be, and the income redistribution is achieved alongside real economic costs. If redistribution is a gain, it must be compared with its associated costs.

Following a formal solution to both a one-sector and a two-sector model of quotas, where in the two-sector case quotas are imposed on only one sector, a number of simulations are presented. The simulations are an arithmetical exercise conducted to give order-of-magnitude estimates of the social cost of quota-induced income redistribution. The main conclusion is that without skill bumping quotas are expensive means of redistributing income.

With skill bumping, quotas have the potential of redistributing income at costs that appear trivial, but there is a rub. Skill bumping

presumes that workers are upgraded into better paying jobs than they would otherwise hold. It is important, however, that bumping cannot increase a worker's productivity as his product is viewed by his employer. If it did, bumping would occur in any case and would not require the impetus of a quota. Skill bumping can, however, lower the productivity of a bumped worker and even if it does, private incentives for bumping can be maintained. It is not necessary that a bumped worker's productivity be impaired. Indeed, private incentives are that it not be. But if it is, the question of the degree of impairment is paramount. For example, in the simulations presented, when impairment does not occur, the social costs of income transfers are trivial. But both the extent of the income transfer and social costs are sensitive to the degree of impairment, and in general costs appear to rise relative to the transfer as impairment increases. In the examples offered, costs amount to only 1 or 2 percent of minority income gains with zero impairment, but increase to 50 percent as impairment rises from zero to 25 percent of the bumped worker's product.

Social costs relative to income transfers are not the only issues of interest. Quotas not only redistribute income from majority to minority populations, but affect the income distribution within each population as well. In particular, the relative wage of skilled to unskilled workers can be affected. These effects can differ between minority and majority populations, and the qualitative nature of these effects is indeterminate, a priori. Aside from pure equity considerations, effects on skilled/unskilled wage ratios are important inasmuch as they signal gains achieved through the acquisition of skills. Protagonists of affirmative action have argued that quotas for skilled minority workers create incentives for increasing minority skills and therefore provide longer run incentives for eliminating the skill component of majority/minority income differentials. That these effects are in fact ambiguous is established by examination of a very simple one-sector model in which quotas only redistribute income without social costs. It may be that quotas do increase returns for minorities to acquire skill, but the theoretical case for this--without more empirical evidence than is now available--appears no stronger than the contrary

case in which quotas create incentives for long-run exacerbation of minority/majority skill differences.

In sum, the analysis presented here suggests that the antagonists' fears of quota programs with high social costs alongside widespread sacrifices of employment standards may be ill-founded. On the contrary, it is the flexibility of standards that can dampen costs. Similarly, the protagonists' optimism of long-run incentives toward skill convergence may also be ill-founded. As in many other cases, effects of quotas seem to be largely an empirical question.

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## I. INTRODUCTION

Reactions to employment quotas are seldom mild. Protagonists argue that they create incentives for minority/majority convergence in skill distributions. In their view, quotas, unlike simple transfers, promote income equality through equality of worker productivities. Antagonists argue that quotas are socially costly; they result both in sacrificing standards and inefficient use of resources. My objective is to present an analytical framework showing that in this, as in most cases, effects are not necessarily what they seem.

I assume that quotas are target minority/majority employment ratios for skilled or highly paid occupations. I also assume that in covered occupations, firms must pay the same wage to all workers who have the same job title. The presumption is that target ratios are not easily satisfied with available supplies of skilled minority workers, so that quotas create an effective shortage of these workers. Associated with this shortage, there is an incentive to distinguish between demographic (color or sex) and skill dimensions of the quota. This incentive is indeed one to "sacrifice standards," to pass and pay unskilled minority workers *as though* they are skilled, because in so doing firms acquire the option of hiring additional skilled majority workers. I call this process of passing (and paying) unskilled minority workers for skilled "skill bumping." Failure to recognize opportunities for skill bumping can result in serious overestimates of all program effects and, in particular, social costs will be overstated. In fact, skill bumping can convert a program with extreme effects into a dampened one of relatively inexpensive income transfers. It is also true that effects on minority and majority incentives for acquiring skills are ambiguous. The case for assuming that quotas result in long-run convergence between minority and majority skill distributions appears no stronger than the case for assuming that quotas result in skill divergence.

The historical roots for modern emphasis on employment ratios are derived from the 1964 Civil Rights Act which proscribes discrimination on the basis of race, color, sex, creed, or national origin. Executive

Orders #11246 (1965) and #11375 (1967) put teeth in this legislation, at least for federal contractors: Each contractor must not discriminate among those currently employed and must institute an affirmative action plan designed to integrate its work force by providing more jobs for minorities and women.

There is a real question of the effect of these statutes. It is clear that with respect to blacks both earnings and occupational status have improved relative to whites in the period since 1964. But relative wages were rising before then, and the extent of recent gains that can be attributed to antidiscriminatory mandates is unclear. At some future time we may have data good enough to identify these effects, but this information is not now available. In this report, I try to deduce upper bounds for effects by considering affirmative action in its most extreme form as being an employment quota.

The most novel feature of a quota is that the employer's perspective is shifted from that of the individual worker to groups of workers in which the minority/majority ratio has a lower bound. In a quota-constrained competitive equilibrium, instead of a worker's wage being the value of his own (marginal) product, it will be the total product of his group divided equally among its members. To the extent that quotas affect wages and employment, a number of rationing problems are introduced. For example, if a quota restricts employment in covered occupations because relevant minority/majority population ratios fall short of quoted ratios, then surplus majority workers will seek admission to occupations with limited membership. Within covered units, those members whose productivity exceeds the average would prefer to exclude those with below average product, while those of lower ability would seek entry. Employers would also have incentives to bypass quota constraints that from their perspective increase costs of production. My approach to these rationing problems is to emphasize the resolution to one (through skill bumping) and to ignore others. The effect is illustrative of overall dampening that occurs when markets respond to incentives implicit in programs that would generate unequal pay for similarly skilled workers, and nothing in current statutes prevents skill bumping if it is detectable and detected.

## I. INTRODUCTION

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The next two sections of this report consider static implications for a world with only two labor skill categories, and the last section, based on an assumption of continuous skill, speculates about dynamic implications for long-run trends in minority/majority skill differentials. The maintained assumption is that average skill levels are lower for minority than for majority populations. Because of Executive Orders #11246 and #11375, the quota problem is a two-sector one in which only federal contractors are covered. Nonetheless, coverage may in principle be extended to the entire economy, and many of the simpler and more obvious points can be made in the context of full coverage. Section II briefly considers an economy-wide quota. Section III extends the analysis to two sectors, and effects are developed more fully in the two-sector than in the one-sector treatment. This is because the one-sector case is only a limiting form of the two-sector one and because the discussion of social costs is richer with two sectors. Section III also contains results of numerical simulations based on a variety of assumptions about the scale of the program and effects of skill bumping. Section IV explores implications of quotas on returns to acquiring skill. In this case, skill is continuously variable and quota coverage includes the entire economy.

## II. ONE SECTOR, TWO SKILLS

Consider an economy-wide quota requiring that for each skilled majority worker hired, at least r skilled minority workers must also be hired. The aggregate production process is assumed linearly homogeneous and depends on two types of labor, skilled and unskilled. The production function is

$$x = f(a, b) , \tag{1}$$

where a refers to unskilled labor and b to skilled labor. There are two populations, the minority and the majority, and labor is presumed of fixed supply. The minority/majority population ratio is $\pi$ and the minority/majority skilled labor ratio is $\pi_b$. The presumption is that $\pi > \pi_b$ and $r > \pi_b$.

Labor is assumed freely mobile between firms and discontinuities are ignored, so that if $r \leq \pi_b$, the quota would cause some reshuffling of workers between firms, without any substantive effect.

The possible preexistence of labor market discrimination complicates the analysis. There is the question of who discriminates and how discrimination affects specification of the production function. The now classical taste-based treatments of Becker (1971) and Arrow (1973) offer some insight, and their dichotomy between employer and employee discrimination is useful.[1*]

Employers who discriminate are seen acting *as though* minority workers are less productive than similarly skilled majority workers. Although problems of obtaining a stable competitive equilibrium are well known, if an equilibrium is reached, it is *as though* aggregate stocks of skilled and unskilled labor are somehow reduced from their physical equivalence, in which minority and majority workers have equal weight to value equivalents where minority workers are given lower weight in computing aggregates. In this vein, it does not matter whether physical

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*Footnotes are located at the end of sections.

productivities are actually lower for minorities or whether the relevant economic actors only act *as if* they are lower, since, in a one-sector model the distinction between physical output and its value is meaningless. Independently of distributional concerns, discrimination lowers the perceived value of aggregate product and is socially costly relative to a "better world." If tastes for discrimination vary among employers, those with little discriminatory intent have a comparative advantage in hiring minorities and would do so in the absence of quotas. Because quotas distribute the skilled minority among firms in proportion to skilled majority employment, average levels of discrimination increase with quotas and thereby increase social costs.

With employee discrimination, workers of different groups are seen as having to be bribed to associate, and tendencies toward segregated production emerge. These may be partly offset, as in the linear homogeneous case, if the production process is such that skilled and unskilled workers are mutually enhancing, that is, $f_{ab} > 0$. If minority/majority population and skilled labor ratios, respectively, $\pi$ and $\pi_b$ were equal, employee discrimination would result only in completely segregated production. But since $\pi_b < \pi$ is assumed, mutual complementarities create incentives for integration. If tastes for discrimination vary among workers, those with relatively little distaste for association with other groups would have a comparative advantage in integrated production, and, as with employer discrimination, a quota that forces uniform integration would increase the average level of discrimination.

In the absence of a quota, I assume the unskilled minority wage is some fraction, $\alpha_a (\leq 1)$, of the majority wage. The corresponding ratio for skilled wages is $\alpha_b (\leq 1)$. If $\alpha_a = \alpha_b = 1$, there is no discrimination. Quotas require equal pay for equal work in covered occupations. In an associational unit consisting of one majority and r minority workers, if the perceived product of a majority worker is $f_b$ and for a minority worker is $\alpha_b f_b$, then the wage, $w_b$, paid to each member of the group will be $f_b(1 + \alpha_b r)/(1 + r)$. This averaging within groups can be viewed as a simple majority to minority transfer.[2] Full employment is assumed for the pre-quota equilibrium, so that the quota with $r > \pi_b$ creates a surplus of skilled majority workers.[3]

Two questions arise: (1) What mechanism exists to ration the restricted number of jobs for skilled majority workers among the available supply? and (2) What becomes of the displaced majority workers? Assume temporarily that the rationing problem is solved by lottery and that displaced workers become unemployed.

Three effects are immediate. First, since aggregate employment has fallen, aggregate output must fall.[4] Most probably, average majority income would fall.[5] Second, the aggregate skilled/unskilled employment ratio is reduced, and the productivity of skilled labor will rise relative to the productivity of unskilled labor. The minority is unskilled labor intensive, so this effect is to *lower* the average income of minority workers.[6] The third effect is the transfer from those skilled majority workers who remain employed to skilled minority workers noted above.[7] This transfer component does not exist in the absence of discrimination.

Now, consider the question of what happens to displaced majority workers. Firms have two options for employing them. Either they can be renamed "unskilled" or some unskilled minority workers can be classified as "skilled" to permit use of the displaced majority as skilled workers.

If displaced majority workers are bumped (dumped?) into unskilled jobs, their productivity may be impeded. In the absence of the quota, the productivity of skilled relative to unskilled majority workers is $f_b/f_a$. Let $\Theta_1$ denote the productivity of a skilled majority worker, dubbed "unskilled," relative to an unskilled majority worker. It seems reasonable to expect that $1 \leq \Theta_1 \leq f_b/f_a$. The lower limit holds if skilled workers, called unskilled, are no more productive than unskilled workers. In this limit, the quota tends toward a reduction in average income for both the minority and majority groups.[8] The downward tendency in minority earnings is ameliorated through the redistribution implicit in "averaging" costs of discrimination among all skilled workers.

In the case where the upper bound ($\Theta_1 = f_b/f_a$) holds--where the productivity of displaced skilled majority workers is unaffected by reclassification--the quota has no effect on aggregate output.[9] Any effect on earnings is restricted to a majority to minority transfer

among skilled workers, dubbed skilled, implicit in averaging costs of discrimination.

The second alternative consists of renaming some unskilled minority workers "skilled" and hiring them to work in association with otherwise displaced majority workers. This case is not the same as the previous one of bumping skilled majority workers into unskilled job classes. First, to the extent that reclassification impedes worker productivity, there is no necessary relation between the loss associated with bumping downward as opposed to bumping upward. Second, since only skilled job classifications are covered by the quota, the equal-pay-for-equal-work constraint is not binding for downward bumping but is for upward bumping. Third, if bumping impairs productivity, upward bumping increases the aggregate skilled/unskilled employment ratio while downward bumping reduces it.

Define $\Theta_2$ as the productivity of an unskilled minority worker, called skilled, relative to a skilled majority worker. It seems reasonable to suppose $0 \leq \Theta_2 \leq f_a/f_b$. If the lower bound holds, a bumped minority worker loses all of his potential product through reclassification. If the upper limit holds, there is no discrimination and worker productivity is unaffected by reclassification. It may appear surprising that this upper limit is not set to unity. Note, however, that regardless of how much a firm pays a worker, it always has an incentive to use him where his productivity is greatest. That unskilled workers are classified as unskilled in the absence of the quota is proof in and of itself that their productivity cannot be enhanced by simple reclassification. If each member of an associational unit consisting of one skilled majority worker and r bumped minority workers receives a wage of $w_b'$ then the cost per unit productivity of members of this unit is

$$\frac{w_b'(1 + r)}{1 + r\Theta_2} .$$

If workers in composites consisting of skilled minority and skilled majority workers each receive a wage, $w_b$, costs per unit or productivity are $w_b(1 + r)/(1 + \alpha_b r)$. Since firms would have the option of hiring

skilled labor from either of the two sources, an equilibrium condition is

$$w_b = w'_b \frac{1 + \alpha_b r}{1 + \theta_2 r} . \qquad (2)$$

A maintained assumption is that $\theta_2 \leq \alpha_b \leq 1$,[10] so that workers in the upwardly bumped composites earn lower wages than in composites in which all workers are skilled.

If the lower bound, $\theta_2 = 0$, holds, the aggregate skilled/unskilled labor ratio must have increased. This effect alone would tend to increase (unskilled labor intensive) minority incomes and to reduce (skilled labor intensive) majority incomes. In addition, two income transfer effects tend toward increased minority incomes. The first is the often noted averaging of costs of discrimination within fully skilled composites while the second would be a transfer in mixed composites from skilled majority to unskilled minority workers. The second transfer occurs independently of discrimination, but the first depends on the existence of discrimination.

In the upper bound, $\theta_2 = f_a/f_b$, there is no discrimination, and worker productivity is insensitive to reclassification. In this case the quota is socially costless, and its only effect is to transfer income within mixed composites from skilled majority to unskilled minority laborers. Intermediate cases, $0 < \theta_2 < f_a/f_b$, are qualitatively similar to the lower bound, but effects are less pronounced.

This essentially exhausts the analytical possibilities in a one-sector/two-skill model. The important point is that rationing problems emerge and that skill bumping offers a partial resolution. In fact, skill bumping introduces its own rationing problem: What determines who will be bumped? A world with bumping establishes a number of wage differentials among otherwise similar workers. For example, with downward bumping of skilled majority workers, wages of those bumped are not necessarily equal to wages of those skilled majority workers who are not bumped. With upward bumping of unskilled minority workers, wages of those bumped exceed wages of those not bumped and wages of skilled

majority workers working in homogeneous composites exceed wages in heterogeneous composites.

Skill bumping offers an option for utilizing workers who would otherwise be unemployed by simple reclassification of job titles. If worker productivity is not reduced by reclassification, skill bumping changes a socially costly program into a costless one involving only income transfers. If reclassification impedes productivity, although social costs remain, they may be dramatically reduced.

Before turning to the two-sector specification, consider the question of which form of skill bumping is most likely and whether the two forms might coexist. Unless rationing mechanisms are specified along with the exact relation between bumping and productivity change, no solution is possible. If productivity is not affected by bumping, either will fully eliminate direct output effects. In this case we might suppose that skilled majority workers would prefer to be bumped into unskilled job classes, since transfer payments to minorities are not required as they would be if unskilled minority workers were bumped into skilled job classes. This is, of course, an artifact of the assumption that the equal-pay-for-equivalent-job-title constraint is not extended to unskilled jobs. If it were, the total product of all workers bearing the same title would be shared equally. In this case displaced skilled majority workers would prefer downward bumping only if the actual ratio of unskilled to skilled workers within the unskilled job classification fell short of the quoted ratio that would hold with the upward bumping alternative.

At the opposite extreme, if productivities of bumped workers satisfy indicated lower bounds, downward bumping is preferred by those directly involved if $f_a > f_b/(1 + r)$. Otherwise, upward bumping is preferred.

If proportionate productivity effects are symmetrical, that is, if downward bumping reduces a skilled laborer's productivity by a given fraction, and upward bumping reduces an unskilled worker's productivity by the same fraction, then upward bumping is clearly preferred. First, skilled workers are presumed more productive than unskilled workers. Second, upward bumping reclassifies fewer workers. A maintained

assumption is that the quoted ratio $r < 1$, that is, the quota refers to a minority population. Thus, for each skilled majority worker who would be bumped into unskilled job classes, only r unskilled need be bumped into skilled classes.

In the two-sector discussion, only upward bumping is considered.

## FOOTNOTES

[1]At the outset, note that there is a demonstration effect argument favoring quotas that I do not address. The presumption of this argument is that employers have biased expectations of minority worker productivity. Quotas increase integration in skilled occupations, and gains are realized as employer fears of adverse effects are not substantiated. The import of this view depends on the extent to which perceived productivity of minority workers can be enhanced by quotas. It is possible that historically based estimates of minority productivity have not fully incorporated recent gains, in which case quotas might speed adjustment.

[2]It is noteworthy that as long as the question of skill acquisition, of converting unskilled into skilled workers, is ignored and workers are presumed of fixed supply, all wages are rents. The potential for employee discrimination will not affect factor use and will therefore have no effect on observed wages: Only their psychic equivalents to workers are affected by integration. Since the discussion refers to observed wages and incomes, the presumption of observed discrimination refers either only to employer discrimination or to a form of employee discrimination in which labor productivities, *as seen by employers*, are affected by integration.

[3]The number of skilled majority workers who can find employment in association with skilled minority workers is $1/r$ times the number of skilled minority workers.

[4]Ignoring any increased costs of association arising from increased average discrimination, the linear approximation to the fractional change in aggregate output is $dy/y = s(db/b)$, where s is the skilled labor's aggregate share and

$$\frac{db}{b} = \frac{\pi_b - r}{r(1 + \pi_b)} < 0$$

is the fractional change in employment of skilled labor.

[5]In principle, total majority income can actually be increased by a quota. Let $b_o$ refer to the aggregate number of skilled workers, while $b_1$ is the number of these workers who are members of the majority population, where $s_b = b_1/b_o$. Corresponding numbers for unskilled workers are $a_o$ for the aggregate and $a_1$ for the majority with $s_a = a_1/a_o$. By assumption, the majority population is skill intensive, so $s_b > s_a$. Without the quota total majority income, $w_m = a_1 w_a + b_1 w_b = a_1 f_a + b_1 f_b$. The quota reduces total skilled employment and the full reduction is realized by majority workers, that is, $db = db_1$. We have $dw_m/db = a_1 f_{ab} + f_b + b_1 f_{bb}$. Since $a_o f_{ab} + b_o f_{bb} = 0$ and $\sigma = f_a f_b / x f_{ab}$, for linear homogeneous functions in which $\sigma$ is the substitution elasticity, the change in majority earnings can be expressed as

$$\frac{dw_m}{db} = f_b \left(1 - \frac{(s_b - s_a)k}{\sigma}\right),$$

where k is unskilled labor's share of aggregate product. Since $db < 0$, $dw_m > 0$ if $\sigma - (s_b - s_a)k < 0$.

[6]For this component of the fractional minority income, we have

$$\frac{dw_n}{w_n} = (1 - k)\frac{dw_a}{w_a} + k\frac{dw_b}{w_b},$$

where $w_n$ refers to total minority earnings, k to skilled labor's share of $w_n$, and $w_a$ and $w_b$ are the wage rates for unskilled and skilled labor, respectively.

By presumption of competitive discriminatory equilibrium,

$$w_a = \alpha_a f_a \quad \text{and} \quad w_b = \alpha_b f_b .$$

Further,

$$\frac{b}{w_a}\ \frac{\partial w_a}{\partial b} = \frac{s}{\sigma} \quad \text{and} \quad \frac{b}{w_b}\ \frac{w_b}{\partial b} = \frac{-(1 - s)}{\sigma} ,$$

where $\sigma$ is the substitution elasticity between a and b and s is skilled labor's share of aggregate output. The change in average minority income can be expressed as

$$\frac{dw_n}{w_n} = \frac{(s - k)}{\sigma}\ \frac{db}{b} ,$$

and since $s > k$ and $db/b < 0$, it follows that

$$\frac{dw_n}{w_n} < 0 .$$

[7]This component is

$$\frac{dw_n}{w_n} = \left(\frac{k}{1 + r}\right)\left(\frac{1 - \alpha}{\alpha_b}\right) .$$

[8]The effect on minority incomes is obvious since the minority group is intensive in unskilled labor (relative to the population aggregate), and the relative wage of unskilled labor falls with the change in factor ratios. However, the effect on minority incomes is of second-order importance in comparison with the effect on majority incomes where the loss is approximately $f_b - f_a$ for each worker transferred from skilled to unskilled employment. But the direct loss to majority workers from transferring, from earning $f_b$ to $f_a$, is partially offset by the rising relative wages of skilled labor. It is possible that the second effect dominates the first. This would be true if $(s_b - s_a) > [\sigma - (s_b - s_a)k]\ (1 - f_a/f_b)$, which seems unlikely.

[9]I omit any possible increased costs of discrimination resulting from increased minority/majority association.

[10]Notice that $\Theta_2$ is the product of the relative productivity of an unskilled minority worker classified as skilled (to a skilled majority worker) and the discrimination coefficient against these minority workers. Although it is not clear whether $\alpha_a$ or $\alpha_b$ is the appropriate discrimination coefficient or whether a wholly new value should be introduced, it does seem reasonable to expect $\Theta_2 < \alpha_b$, since $\Theta_2 = \tilde{\alpha}_b \tilde{f}_a / f_b$, where $\tilde{\alpha}_b$ is the "appropriate" discrimination coefficient, and $\tilde{f}_a (\leq f_a)$ is the productivity of a reclassified unskilled worker.

## III. TWO SECTORS, TWO SKILLS

In deference to Executive Orders #11246 and #11375 which exert the clearest pressures for "good faith" compliance to affirmative action on federal contractors, I assume that there are two sectors, one covered, the other not. The assumption is that in the pre-quota state the economy is in a fully competitive equilibrium in which workers receive wages equal to values of marginal products and there is no discrimination.

I ignore the possible preexistence of discrimination (1) because qualitative changes do not depend on discrimination, (2) because I frankly do not know how to fully incorporate discrimination into measures of social costs, and (3) because effects of averaging or sharing costs of discrimination within skill composites is obvious a priori. Whether discrimination exists or not, a quota's main effect is to increase the cost of skilled labor to covered firms, and all other changes derive from this initial impetus. Without discrimination, costs of skilled labor are increased through the quota-constrained reduced supply of skilled labor. An important assumption of the two-sector model is that workers retain employment options in the uncovered sector such that a skilled majority worker will not opt for covered sector employment at a wage below his uncovered sector alternative. With employer discrimination in which minority workers receive lower wages than majority workers, the quota's equal-pay constraint requires that skilled minority workers receive wages at least as high as the uncovered sector's skilled majority wage, thereby increasing the cost of skilled labor to the covered sector. With employee discrimination, skilled majority workers in the covered sector are employed by quota-constrained integrated firms and must receive wages above their uncovered sector alternative where options for segregation are maintained. Again, this effect is to increase the cost of skilled labor in the covered sector.

The second reason for ignoring discrimination is that it completely confounds measurement of social costs. At best, discussions of social cost are conceptually difficult, and the measures I use are similar in

principle to those commonly used and are therefore subject to the standard criticism, Whose cost? Relative to what? The traditional gambit of the typical consumer-producer seems peculiarly arid in a discriminatory environment in which one's reaction to another is the driving mechanism. There are, of course, some obvious points involving social costs in a discriminatory environment. If as in the standard cases of employer and employee discrimination, integration is costly, then to the extent that quotas increase integration, they increase costs of production. However, if demonstration effects are positive, increased integration will have a socially beneficial effect. It is tempting to say that if discrimination is perceived as quantitatively important, measures of cost presented in this section should be ignored. Be that as it may. The common antagonist's argument is that quotas unnecessarily increase costs of income transfers. It seems reasonable to examine this argument on grounds most favorable to it, that is, for a world without discrimination.

The third reason for ignoring discrimination is that in addition to a vast complication of notation, without compensating insight into effects, many of the changed effects resulting from preexistent discrimination can simply be "dubbed in" ex post. The transfer implicit in averaging costs of discrimination within skilled composites is an obvious example.

The covered sector product is

$$x = f(a_1, b_1) \ , \tag{3}$$

where x is output and $a_1$ and $b_1$ refer, respectively, to employment of unskilled and skilled labor. Similarly, the uncovered sector output is

$$y = g(a_2, b_2) \ , \tag{4}$$

where $a_2$ and $b_2$ denote employment. The uncovered sector is taken as the numeriare (unit product price), and the price of the covered sector's product is p. Both production functions, $f(\cdot)$ and $g(\cdot)$, are

assumed linear homogeneous and production is subject to fixed-input aggregates. Thus,

$$a = a_1 + a_2 \quad \text{and} \quad b = b_1 + b_2 \ .$$

Labor is assumed freely mobile so that equilibrium conditions are

$$w_a = w_{a1} = w_{a2} \ , \tag{5}$$

$$w_b = w_{b1} = w_{b2} \ , \tag{6}$$

$$\phi(p) = x = f(a_1, a_2) \ . \tag{7}$$

The first is that the wage of unskilled labor, $w_a$, is the same in the covered as in the uncovered sector. In the covered sector this wage $w_{a1} = pf_a$, and in the (*numeraire*) uncovered sector this wage $w_{a2} = g_a$, where $f_a$ and $g_a$ in the standard notation refer to marginal physical productivities. The second condition is that the wage of skilled labor is the same in the two sectors where $w_{b1} = pf_b$ and $w_{b2} = g_b$. The third condition is that the demand for the covered sector's product, $\phi(p)$, equals supply, $f(a_1, b_1)$.

The quota requires that in the covered sector for each skilled majority worker hired, there must be at least r skilled minority workers and that, within firms, skilled minority and skilled majority workers receive equal wages. If in the economy there are a total of n skilled minority workers, this constraint is that, without bumping, employment of skilled labor in the covered sector cannot exceed

$$b_1'(r) = n(1 + r^{-1}) \ , \tag{8}$$

where n refers to the available supply of skilled minority workers, and n/r is the maximum number of majority workers who can be assigned to them. In the pre-quota equilibrium, if skilled employment is in the

covered sector, $b_1^o$, the quota will have no direct effect unless $b_1' < b_1^o$. If $r_o$ is defined such that $b_1'(r_o) = b_1^o$, then $r_o$ represents the minimum or threshold level at which the quoted ratio becomes binding. If the actual quota $r < r_o$, there is no effect, while if $r > r_o$, the quota represents an effective deterrent to skilled employment in the covered sector.

*Without skill bumping*, an effective quota operates as an exogenous shift of skilled labor from the covered to the uncovered sector. The quota vitiates equilibrium condition (6) leaving conditions (5) and (7) intact. Since the quota shifts skilled workers from the covered to the uncovered sector, the wage of skilled labor must rise in the covered relative to the uncovered sector. Further, covered sector output must fall and, correspondingly, product price must rise.[1] Notice that because the uncovered sector is taken as the *numeraire*, the assumption of a linear homogeneous production relationship is an assumption that wages (marginal products) depend only on factor ratios, and therefore if the wage of skilled labor falls in that sector, the wage of unskilled labor must rise and vice versa. At this point, the model is formally identical with that of Johnson-Miezkowski (1970) who analyze effects of unionism viewed as an exogenously given change in relative wages between union and nonunion sectors for the two-sector-linear-homogeneous case. This and similar models have also been analyzed in the international trade literature, so that results are neither new nor surprising. For this reason, I only sketch the nature of quota effects and relegate exact solutions for linear derivatives to the first section of the appendix.

The standard results indicate a variety of possible outcomes for the post-quota equilibrium, but several possibilities can be ruled out if the covered sector is skill intensive. Because federal contractors include defense and educational sectors, a presumption that the covered sector is skill intensive seems reasonable.

The movement from the pre- to the post-quota equilibrium can be described as if it is accomplished in two stages. The first refers to effects associated with the transfer of skilled workers to the uncovered sector, holding the sectoral distribution of unskilled labor at pre-quota

levels. Because the first step will normally result in different wages for unskilled labor in the two sectors, the second step involves migration to the sector having the higher wage. The second-stage movement will increase unskilled wages in the sector of origin and lower them in the recipient sector until equality is reestablished. Throughout these two steps, the supply-demand relation, Eq. (7), is assumed to be continuously satisfied.

The following partial elasticities are easily deduced:

$$\frac{Ew_{a1}}{Eb_1} = \frac{s_1(\sigma_1 + \eta)}{\sigma_1 \eta} \; ; \quad \frac{Ew_{a2}}{Eb_1} = -\frac{b_1}{b_2}\,\frac{s_2}{\sigma_2} \; ; \quad \frac{Ew_{b1}}{Ea_1} = \frac{1 - s_1}{s_1}\,\frac{Ew_{a1}}{Eb_1} \; ,$$

where $Ez = dz/z$; $s_1$ and $s_2$ are product shares of skilled labor, respectively, in the covered and uncovered sectors; $\eta$ is the covered sector price elasticity of demand; and $\sigma_1$ and $\sigma_2$ are substitution elasticities in the respective sectors between skilled and unskilled labor.

The first-order effect is to increase $w_{b1}$, to reduce $w_{b2}$, and to increase $w_{a2}$. The effect on $w_{a1}$ is ambiguous since declining (marginal) factor productivity is contradicted by rising product price. If $\sigma_1 + \eta < 0$, the substitution effect dominates and $w_{a1}$ will fall. Otherwise, if $\sigma_1 + \eta > 0$, it will rise and migration cannot overturn the initial impetus of an increased wage in both sectors.

If $Ew_{a1}/Eb_1 > Ew_{a2}/Eb_1$, unskilled labor will migrate to the uncovered sector, because $db_1 < 0$, but the extent of this migration must in any case be such that the skilled/unskilled labor ratio is increased in the uncovered sector. This can be seen by considering radial increments in both skilled and unskilled labor in the uncovered sector that maintain factor ratios at pre-quota levels. Such movements would hold wages in the uncovered sector constant. But since the covered sector is skill intensive, these movements would increase the skilled/unskilled labor ratio in the covered sector while reducing output and increasing product price. The effect would be to increase unskilled wages in the covered sector, so that while migration to the uncovered sector may result, the extent of this migration could never maintain or lower the skilled/unskilled labor ratio in that sector.

A general result of the assumption that the covered sector is skill intensive is, therefore, that in the post-quota equilibrium wages of unskilled labor must rise and wages of skilled labor in the uncovered sector must fall. Three cases are identified to determine effects on wages of skilled labor in the covered sector.

In the first case, $\sigma_1 + \eta < 0$, the initial effect is to reduce $w_{a1}$ so that labor migrates to the uncovered sector. As unskilled labor leaves the covered sector, the second-stage effect is to reduce $w_{b1}$. And if enough migration occurs, such that the post-quota equilibrium skill/unskilled labor ratio exceeds the pre-quota level, it is possible--but not necessary--that the second-stage effect dominates the first. Therefore, with $\sigma_1 + \eta < 0$ the effect of the quota on the wages of skilled labor in the covered sector is ambiguous.

The second case corresponds to $\sigma_1 + \eta > 0$ with $Ew_{a1}/Eb_1 > Ew_{a2}/Eb_1$, so that unskilled labor migrates to the uncovered sector. In this case, since $\sigma_1 + \eta > 0$, migration ($da_1 < 0$) increases $w_{b1}$ and reinforces the first-stage effect.

The third case corresponds to $\sigma_1 + \eta > 0$ with $Ew_{a1}/Eb_1 < Ew_{a2}/Eb_1$, so that migration is to the covered sector ($da_1 > 0$). In this case, the second-stage effect is to reduce $w_{b1}$, but this cannot offset the initial impetus. Assume that the extent of migration is sufficient to return $w_{a1}$ to its initial level. At this point, since both the first- and second-stage effects are to reduce the skilled/unskilled employment ratio in the covered sector, the productivity of unskilled labor is diminished and must therefore have been fully compensated by an increased product price. Further, this movement clearly overstates the extent of migration (to the covered sector) since at this point $w_{a2} > w_{a1}$. And, with $\sigma_1 + \eta > 0$, reductions in $a_1$ from this level will further increase $w_{b1}$.

In sum, if the covered sector is skill intensive, a quota must increase the wage of unskilled labor and reduce the wage of skilled labor in the uncovered sector. A sufficient, but not necessary, condition for an increased wage of skilled labor in the covered sector is that $\sigma_1 + \eta > 0$.

If the wage of skilled labor rises in the covered sector, a quota would not only increase minority income relative to majority income but would increase the wage of every member of the minority population. Winners would include all unskilled laborers, skilled minority workers, and those skilled majority workers who are lucky enough to find jobs in the covered sector. Losers would be skilled majority workers in the uncovered sector.[2]

There is an obvious rationing problem associated with this solution inasmuch as otherwise equally skilled majority workers would receive unequal pay in the two sectors. If the equal-pay constraint is not enforced for skilled workers in covered-sector firms, this problem can be resolved through reverse discrimination as skilled minority workers extract the rent implicit for majority laborers who work alongside them. Alternatively, skill bumping offers a way of ameliorating if not fully eliminating the rationing problem.

*With skill bumping*, suppose that the productivity of an unskilled worker is dubbed skilled in some proportion, $\Theta$, of the productivity of an unskilled worker who is classified as unskilled. That is, reclassification of an unskilled worker results in the loss of the fraction, $1 - \Theta$, of his product where $0 \leq \Theta \leq 1$. A firm in the covered sector has two options for hiring skilled labor. It can form composites consisting of r skilled minority workers for each majority worker. In this case, the equal-pay-for-equal-job-title constraint requires that workers in these composites each receive $w_{b1}$. Also, if $w_{b2} > w_a$, firms can hire skilled majority workers at their uncovered-sector alternative wage and match them with unskilled minority workers, who having the same job title would receive the same wage. A "pure" skill composite of $1 + r$ workers would cost the firm $w_{b1}(1 + r)$ and its product would be $f_b(1 + r)$. A "mixed" composite would cost $w_{b2}(1 + r)$ and its product would be $f_b + r\Theta f_a$. An equilibrium condition between these alternative sources of skilled labor to the covered sector is, therefore,

$$w_{b1} \leq w_{b2} \frac{1 + r}{1 + r\Theta f_a/f_b} . \qquad (9)$$

The supply of skilled labor from pure composites is $n(1 + r^{-1})$. If at that level of employment, the inequality in expression (9) holds, skill bumping does not pay. Otherwise, the equality holds, and expression (9) with the equality replaces (6), which alongside (5) and (7) constitute the post-quota equilibrium conditions.

Suppose that in the post-quota equilibrium, employment of skilled labor in the covered sector is

$$b_1 = b_1' + \tilde{b}_1 , \tag{10}$$

where $b_1' = n(1 + r^{-1})$ is employment from pure composites, and $\tilde{b}_1$ is the number of skilled majority workers in mixed composites who work alongside and receive the same wage as $r\tilde{b}_1$ bumped unskilled minority workers. Because of the possibility that $\Theta < 1$, this change is as though the number of unskilled workers has been reduced by $r(1 - \Theta)\tilde{b}_1$. The aggregate stock of skilled labor is as before and $b_o = b_1 + b_2$; however, the useful stock of unskilled labor is $a = a_o - r(1 - \Theta)\tilde{b}_1 = a_1 + a_2$, where $b_o$ and $a_o$ are the initial number of workers.

In addition to whatever social costs are generated through inefficient distributions of workers between sectors, there is the loss associated with the diminished useful number of unskilled workers.

The overall effect of a mandated quota requiring r minority workers for each skilled majority worker in the covered sector can be measured as though the quota is varied over the range from 0 to r. Until the ratio reaches $r_o$, there is no effect other than shuffling workers between firms and sectors. As the quota rises between $r_o$ and some level, call it $r_1$, the inequality in expression (9) will hold and skill bumping will not occur. In this range the equilibrium conditions are expressions (5) and (7), and the equations of motion described in the first section of the appendix are satisfied. As noted earlier, in this range the uncovered-sector skilled wage will fall relative to the covered-sector skilled wage so that skill bumping is increasingly attractive. At the point $r_1$, the equality in expression (9) first holds, skill bumping just pays, and thereafter the equilibrium conditions (5) and (7) are

augmented by condition (9) with the equality. In the second section of the appendix, equations of motion are presented where derivatives evaluated at $r_1$ indicate changes in the range from $r_1$ to r in which skill bumping is in effect. Once the equality of expression (9) is satisfied, quota effects with skill bumping can be described as though they consist of three ingredients. The first is the exogenous shift, $db_1'$, of skilled workers to the uncovered sector associated with the increasing quota level. If this were the only ingredient, effects would be identical with those described without bumping. But with bumping, wages of skilled labor in the two sectors are tied by Eq. (9), and the second ingredient is the flow of skilled labor into the covered sector through mixed composites necessary to maintain this tie. Insofar as wages alone are considered, the second ingredient essentially nullifies the first. In the range where skill bumping is in force, the quota is transformed into an income transfer within mixed composites, from skilled majority workers to unskilled minority workers. The nature of the transfer is that the total product of the composite is equally divided among its members. The third ingredient of effects in the skill-bumping range refers to Eq. (9) itself. As the quota increases, an increased wedge is driven between skilled labor wages in the two sectors. Although the tie is binding, it is being loosened as the quota rises between $r_1$ and r.

SOCIAL COSTS

The most obvious cost of a quota with skill bumping stems from the foregone product of bumped unskilled workers. This loss, evaluated at the pre-quota equilibrium, is

$$w_a r(1 - \theta)\tilde{b}_1 , \qquad (11)$$

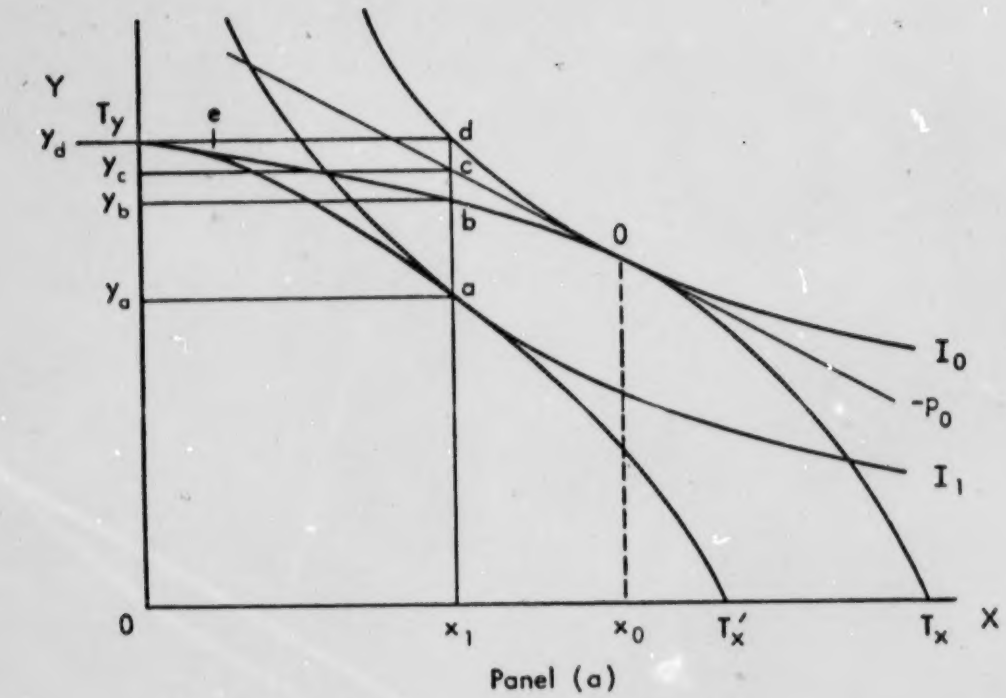
where $r\tilde{b}_1$ is the number of unskilled workers bumped, and $(1 - \theta)$ is that part of their productivity lost by bumping.[3] Other components include the cost of an inefficient distribution of production between sectors and the cost of inefficient allocation of inputs for given product distribution.

Figure 1 illustrates these costs exclusive of the inward shift of the transformation function associated with the direct loss of part of the useful services of unskilled labor. Panel (a) demonstrates the pre- and post-quota equilibria in product space, and panel (b) depicts a transformation into price-quantity space for x, the covered sector's product. The presentations are equivalent to quantities of y (the numeriare) foregone in panel (a), and social costs can be measured from either perspective. The demand function D is taken as a transformation of both welfare indexes, $I_o$ and $I_1$, which follows from the assumption that x has zero wealth elasticity. At the point in input space (the distribution of skilled and unskilled labor between production in the covered and uncovered sectors) where the quota represents a binding constraint, production of x becomes less skill intensive and y becomes more skill labor intensive than would otherwise be true when input distributions coincide with efficient (equal marginal rates of substitution) labor allocation. This inefficient allocation generates an inward shift in the transformation function, from $T_yT_x$ to $T_yT'_x$, or what amounts to a reduced supply of x, from S to S'. The point, e, in panels (a) and (b) represents the point, $r_o$, at which the quota constraint becomes binding and therefore indicates the point of departure between transformation and supply functions for pre- and post-quota situations. In panel (a), the quota constraint results in reduced welfare from index $I_o$ to $I_1$ with the shift from pre-quota equilibrium to the post-quota equilibrium, $y_ax_1$. In terms of the numeriare, the loss in welfare is measured by the distance $y_d - y_a$, which can be decomposed according to:

$y_b - y_a$ = excess cost of production of x at $x_1$, the inward shift in the transformation function,
= area A in panel (b);

$y_c - y_b$ = foregone producer surplus otherwise available with the move from $x_1$ to $x_o$,
= area B in panel (b); and

$y_d - y_c$ = foregone consumer surplus otherwise available with the move from $x_1$ to $x_o$,
= area C in panel (b).



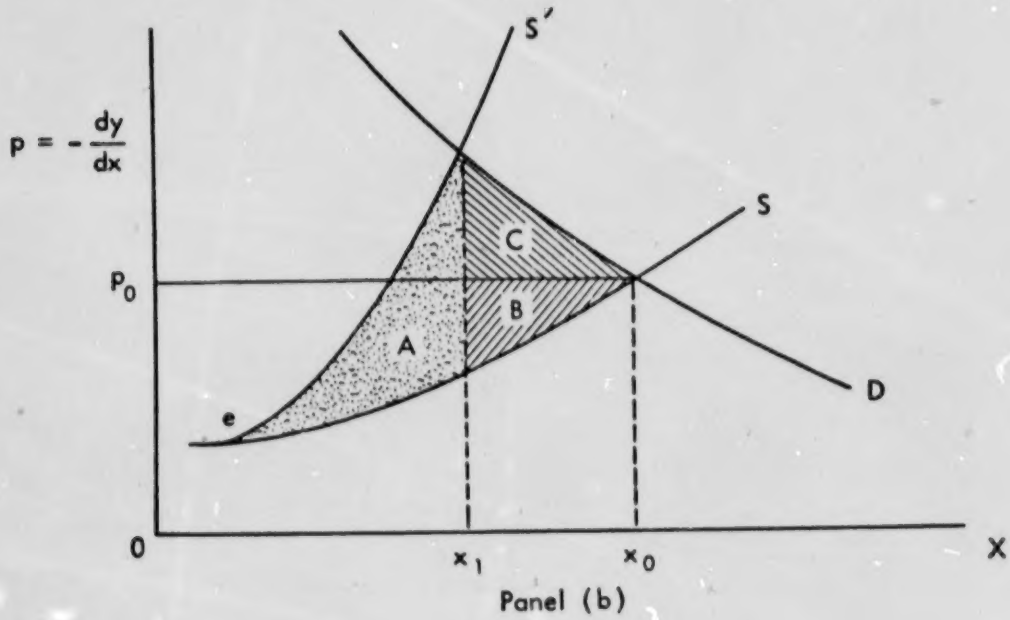




Fig. 1 — An illustration of production adjustments and social costs of employment quotas

The distance $y_d - y_b$ (areas C and B, panel (b)) represents measures of foregone surplus that are standard to this kind of analysis, and approximations of this loss are well known.

Linear approximations to consumer and producer surpluses foregone are

$$y_d - y_c = -\frac{1}{2} p_o x_o \eta^{-1} \left(\frac{dx}{x}\right)^2 , \tag{12}$$

and

$$y_c - y_b = \frac{1}{2} p_o x_o \varepsilon^{-1} \left(\frac{dx}{x}\right)^2 , \tag{13}$$

where $\varepsilon$ refers to the elasticity of supply of x, which in the two-factor (fixed aggregate supply), two-products, linear homogeneous production function case is

$$\varepsilon = \frac{s_1(1 - s_1)\sigma_1 + s_2(1 - s_2)\sigma_2(y/px)}{(s_1 - s_2)^2} .$$

This supply elasticity is derived from the transformation function, $T_y T_x$, by recognizing that $p = -dy/dx = g_a/f_a = g_b/f_b$ and $dx = f_a da_1 + f_b db_1$ (see Lewis, 1975).

The excess cost of producing x with the quota can be measured as the change in y associated with a movement from the point on the contract curve (where the marginal rate of substitution of a for b in production of x is equal to the rate of substitution in producing y) along an isoquant for $x = x_1$ to the post-quota input equilibrium. Linear differentials give

$$dy = g_a da_2 + g_b db_2 \quad \text{subject to}$$

$$dx = f_a da_1 + f_b db_2 = 0 ,$$

$$da = da_1 + da_2 = 0 \quad \text{and} \quad db = db_1 + db_2 = 0 .$$

Substitution of these constraints, dx = da = db = 0, into the differential for production of y gives the excess cost as:

$$y_b - y_a = dy|_{x=x_1} = \bar{g}_a \left( \frac{\bar{f}_b}{f_a} - \frac{\bar{g}_b}{g_a} \right) db_1^* \ . \qquad (14)$$

Notice that since input aggregates are fixed and output of x is fixed at $x_1$, the change $db_1^*$ implies $da_1$ and therefore $db_2$ and $da_2$. The bars over $g_a$, $f_b/f_a$, and $g_b/g_a$ in Eq. (14) refer to mean values over the interval $db_1^*$. Clearly, $y|_{x=x_1}$ is maximized when $f_b/f_a = g_b/g_a$. Let 0 indicate the point in input space satisfying this initial condition and let 1 denote the point in input space that exists post-quota with output of $x=x_1$. Assume that the functions $f_b/f_a$ and $g_b/g_a$ are approximately linear over the interval between points 0 and 1. It follows that

$$\frac{\bar{f}_b}{f_a} \cong \frac{1}{2} \left( \frac{f_b}{f_a}\Big|_o + \frac{f_b}{f_a}\Big|_1 \right) = \frac{f_b}{f_a}\Big|_o + \frac{1}{2} \frac{d(f_b/f_a)}{db_1}\Big|_o db_1^* \ ,$$

and

$$\frac{\bar{g}_b}{g_a} \cong \frac{1}{2} \left( \frac{g_b}{g_a}\Big|_o + \frac{g_b}{g_a}\Big|_1 \right) = \frac{g_b}{g_a}\Big|_o + \frac{1}{2} \frac{d(g_b/g_a)}{db_1}\Big|_o db_1^* \ .$$

Substitution of these approximations into Eq. (14) gives

$$y_b - y_a \cong \frac{1}{2} \bar{g}_a \left( \frac{df_b/f_a}{db_1} - \frac{dg_b/g_a}{db_1} \right) db_1^{*2} \ , \qquad (14a)$$

which can be reduced to

$$y_b - y_a \cong \frac{1}{2} \frac{b_1}{b_2} \frac{px\varepsilon(s_1 - s_2)^2}{(1 - s_1)(1 - s_2)\sigma_1\sigma_2} \left( \frac{db_1^*}{b_1} \right)^2 \ . \qquad (14b)$$

It is important that the change $db_1^*$ in Eqs. (14), (14a), and (14b) measures the difference in covered-sector employment of skilled labor with and without the quota *at the post-quota level of covered-sector output*. Since the quota reduces output, the change $db_1^*$ is smaller than the difference between covered-sector skilled labor employment, pre- and post-quota.

As in the earlier discussion, $db_1$ refers to the difference between skilled labor employment in the covered sector, pre- versus post-quota. This change can be considered as consisting of two parts. The first, $db_1^{**}$, is associated with efficient reduction in x accompanying the quota, while the second, $db_1^*$, refers to the deviation from the cost-efficient locus. For cost-efficient changes, we have

$$dx = f_a da_1^{**} + f_b db_1^{**} \tag{15}$$

subject to the constraint that $f_a/f_b = g_a/g_b$ throughout the range of $da_1^{**}$ and $db_1^{**}$. This gives

$$\frac{da_1^{**}}{db_1^{**}} = \frac{b_1 b_2}{a_1 a_2}\left(\frac{a_1\sigma_1 + a_2\sigma_2}{b_1\sigma_1 + b_2\sigma_2}\right). \tag{16}$$

Since dx is observed, $db_1^{**}$ can be computed from Eqs. (15) and (16), and since $db_1$ is also observed, $db_1^*$ can be computed as a residual, noting that

$$db_1^* = db_1 - db_1^{**} \tag{17}$$

for evaluation of the excess cost of production given in Eq. (14b).

SIMULATION

The impact of quotas is a problem of many ingredients. Incomplete coverage is one. Income redistribution is another. Efficiency of resource use or social cost is a third. Enforcement is an open question. Skill bumping adds its own implications. And, finally, there is the

question of incentives for acquiring skills. Alongside all of these is the question of alternative programs as, for example, subsidized education, whose effects are more direct and therefore more easily interpreted.

In many cases, qualitative effects of quotas can be deduced, but quantitative effects remain illusive. I have simulated a number of quota programs in an attempt to gain insight into quantitative effects. In particular, it seems clear that, on average, quotas will increase minority incomes relative to majority incomes. But if quotas also result in inefficient resource use, there is a question of how social costs compare with the associated income redistribution. To evaluate the efficiency of an income redistribution policy, social costs (changes in aggregate income) are compared with the amount of income being shifted from majority to minority workers.

The calculations reported here consider only one quota policy. It refers to a case in which the minority/majority population ratio is 1/8, which is roughly the U.S. black/white ratio. I consider effects in extreme cases to highlight differences between equilibria with and without skill bumping. Throughout, the quoted skilled employment ratio for the covered sector is presumed equal to the population ratio, 1/8, and the minority/majority relative probability of being skilled is set to 0.2. In all cases the wage of skilled labor is 1.5 times the wage of unskilled labor in the pre-quota equilibrium. Results are reported for two coverage rates and aggregate skill mixtures. In Program #1, the covered sector represents 20 percent of GNP, and skilled labor's share in this sector is 40 percent of product whereas it is 20 percent in the sector not covered. Without bumping, Program #1 would reduce skilled employment in the covered sector by 34.1 percent. Program #2 is even more extreme, and the covered sector accounts for one-half of GNP and skilled labor represents 0.6 of covered- and 0.4 of uncovered-sector product. Without bumping, skilled covered-sector employment would decline by 63.4 percent.

Although these implied factor movements are too large to reasonably estimate changes in wages, social cost, etc. from linear approximations to derivatives, in computing effects, I do just that.[4] At best, these

are arithmetic exercises to give some notion of sensitivity to various parameters, and for this purpose the procedure of inferring large changes from linear approximations seems adequate.

Table 1 reports results without skill bumping where the elasticity of demand for the covered-sector product is assumed equal to -0.5 and unit substitution elasticities between skilled and unskilled labor are assumed for both sectors. As is to be expected, implied wage changes are large in Program #1 and are larger still in Program #2, where both the coverage rate and the share of skilled labor are increased. For Program #1 the nominal skilled wage would actually rise 44 percent in the covered sector but would decline 15 percent in the uncovered sector, while the unskilled labor wage would increase by 4 percent. Program #2 gives impossible results: The covered-sector skilled nominal wage would more than double and the unskilled wage would increase 38 percent. In contrast, the wage of unskilled labor would fall 57 percent. Since a pre-quota relative wage of 1.5 is assumed, Program #2 implies a lower wage for skilled than for unskilled labor in the uncovered sector so that if skill bumping is precluded, "skill jumping" would surely follow. In any case, Table 1 makes one point: Namely, while the potential of quotas to redistribute income is great, without skill bumping the cost of this redistribution is high. Notice that with Program #1, each $1 transferred to minority workers is associated with $1.19 deadweight loss. Stated differently, for each $1 of increased minority income, majority income is reduced by $2.19.

Table 2 contrasts these calculations to effects inferred for the same programs when skill bumping is assumed. The calculations are for four values of $\Theta$, the productivity of an unskilled worker dubbed skilled relative to an unskilled worker who is classified as unskilled. As with Table 1, wage effects are not reported. This is because effects with bumping are essentially known a priori.

Condition (9) can be rewritten as

$$\frac{w_{b1} - w_{b2}}{w_{b2}} = r(1 - \Theta g_a/g_b) \ , \qquad (18)$$

Table 1

ILLUSTRATIVE QUOTA EFFECTS WITHOUT SKILL BUMPING[a]

| Program | Social Costs as a Percentage of GNP | | | Social Costs per Dollar Increase in Minority Incomes ($) | Percentage of Majority/Minority Income Differential Eliminated |
|---|---|---|---|---|---|
| | Excess Costs of Production | Foregone Consumer and Producer Surplus | Total | | |
| Program #1 Covered sector represents 20% of GNP. Skilled labor is 40% of covered-sector and 20% of uncovered-sector product. | .182 | .201 | .383 | 1.19 | 52.4 |
| Program #2 Covered sector represents 50% of GNP. Skilled labor is 60% of covered-sector and 40% of uncovered-sector product. | 2.86 | 7.54 | 10.40 | 5.05 | 222.4 |

[a]The elasticity of demand for the covered-sector product equals -0.5, and the elasticity of substitution between skilled and unskilled labor is 1.0 in each sector. In the pre-quota equilibrium, the relative wage of skilled labor is 1.5. The minority/majority population and quoted employment ratios are 1/8 and the minority/majority relative probability of being skilled is 0.2.

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Table 2

ILLUSTRATIVE QUOTA EFFECTS WITH SKILL BUMPING[a]

| Program | Social Costs with Skill Bumping/Social Costs without Bumping | Share of Total Social Costs | | | Social Costs per Dollar Increase in Minority Incomes ($) | Percentage of Majority/Minority Income Differential Eliminated |
|---|---|---|---|---|---|---|
| | | Direct Loss of Unskilled Labor Productivity from Bumping | Excess Costs of Production | Foregone Consumer and Producer Surplus | | |
| Program #1 | | | | | | |
| $\theta = 0$ | .54 | .93 | .03 | .04 | 1.62 | 21.8 |
| $\theta = .5$ | .29 | .94 | .03 | .03 | .91 | 19.3 |
| $\theta = .75$ | .15 | .94 | .03 | .03 | .49 | 18.0 |
| $\theta = 1$ | .00 | -- | .47 | .53 | .01 | 16.7 |
| Program #2 | | | | | | |
| $\theta = 0$ | .15 | .98 | .00 | .02 | 2.20 | 60.0 |
| $\theta = .5$ | .07 | .98 | .00 | .02 | 1.09 | 55.5 |
| $\theta = .75$ | .04 | .98 | .00 | .02 | .54 | 53.2 |
| $\theta = 1$ | .00 | -- | .28 | .72 | .01 | 50.9 |

[a]The elasticity of demand for the covered-sector product equals -0.5, and the elasticity of substitution between skilled and unskilled labor is 1.0 in each sector. In the pre-quota equilibrium, the relative wage of skilled labor is 1.5. The minority/majority population and quoted employment ratios are 1/8 and the minority/majority relative probability of being skilled is 0.2.

where $w_{b1}$ and $w_{b2}$ refer to skilled wages (post-quota) in the covered and uncovered sectors, respectively, and $g_a/g_b$ is the marginal rate of substitution of skilled for unskilled labor in the uncovered sector. Since in both Programs #1 and #2, r = .125, skilled wages can differ by at most 12.5 percent in the two sectors. Further, since the initial value of $g_a/g_b$ is 2/3, since $g_a/g_b$ increases with the quota, and since $\Theta = 1$ as an upper bound, this difference can be smaller than 4 percent. Wage effects with bumping are small in comparison with effects without bumping.

What is clear in Table 2 is that the income redistribution potential of quotas is smaller with bumping than without. Since bumping is partly redistribution (productivity averaging in mixed composites), this result is not obvious a priori. What is also clear is that social costs and, therefore, the efficiency of the transfer mechanism are very sensitive to $\Theta$. In fact, the results described in Tables 1 and 2 are only a minor part of the calculations performed under a variety of assumptions about system parameters.[5] Although for the cases reported in Table 2, social costs with skill bumping fall short of those without, this is not a necessary result. With small values of $\Theta$, which imply large direct productivity losses associated with bumping, there are cases where social costs are increased by bumping, even though *private* incentives for bumping are maintained. The importance of $\Theta$, the fraction of unskilled labor's product retained after reclassification as skilled, in social cost calculations is evident, since it literally swamps other cost components (for Program #1 it represents 94 percent of costs and 98 percent of costs in Program #2 with $\Theta = 3/4$).

Although the extent of the majority to minority income transfer is restricted by bumping, the transfer is not as sensitive to $\Theta$ as are social costs. Consequently, the efficiency of the income transfer is inversely related to it. In each program, when $\Theta = .75$, there is roughly a \$.50 deadweight loss associated with a \$1 transfer to the minority group, and this loss falls to \$.01 as $\Theta$ increases to 1. In its limiting form, where $\Theta = 1$, skill bumping converts a program that could have dramatic effects on wages and social costs into a program of simple income transfers where social costs are well within ranges of rounding error.

It would be nice if we had either a good a priori rationale for evaluating $\Theta$ or, at least, a means of estimating it, but we do not. It is, of course, true that if reclassification does not change a worker's activities--a rose gardener by any other name--then $\Theta$ is 1. So, essentially, the question of whether $\Theta$ is large or small hinges on (1) whether enforcement officers are willing to insist that worker activities conform to job titles and (2), if they are, whether these officers are capable of detecting divergencies. In any case, in the context of the model presented, private incentives of those directly involved are to set $\Theta$ to 1. This follows from the assumption that worker utility depends only on wages. Job titles and activities within a job play no role. Similarly, firms maximize profit. It is therefore in a firm's interest to maximize each worker's contribution to product irrespective of title, and this incentive is fully compatible with those of workers directly involved. In more general models, where titles and amenities play a role, there should be an excess supply of unskilled candidates for bumping since bumping increases wages of those bumped. In this case, firms would select only the most productive from the applicant pool. That is, firms would select those whose productivity is least impaired by reclassification. Furthermore, if the two-skill dichotomy is generalized to a continuum, we can see that the worker being bumped would be the most productive among those not explicitly covered. It is also likely that $\Theta$ depends on the extent of the skill differential over which a worker is bumped, since the cost of being "out of pocket" should depend on the degree to which one is out of pocket.

## FOOTNOTES

[1]Notice that demand for covered-sector product is presumed independent of income and, therefore, of changes in the distribution of income associated with the quota.

[2]To this point, only nominal wages have been considered. But, since the price of the covered-sector product increases while the price of the uncovered-sector product is not changed, the cost of living

increases and evaluation of real wage effects would have to take this into account. Notice that demand for the covered-sector product is assumed independent of income. This convenient assumption is necessary for unambiguous statements concerning social costs. If this assumption of zero aggregate income elasticity of demand for the covered-sector product holds separately for minority and majority groups, it must be true that the minority population spends a larger share of its income on the covered-sector product than does the majority because the majority is assumed to have higher average income. Therefore, the cost of living would increase more for the minority than for the majority. This secondary effect is ignored in subsequent income comparisons.

[3]It may appear that with skill bumping, quotas generate an additional inefficiency since firms in the covered sector can hire skilled labor through either pure or mixed composites. If firms hiring skilled labor from one source were to produce using different factor ratios than firms hiring skilled labor through the alternative route, then output of the covered sector would be less than that which could be achieved with efficient distribution of inputs between firms. In fact, the distribution of inputs between firms is efficient. This can be seen by reference to the assumed linear homogeneity of the production process and the assumed free mobility of unskilled labor. Since all covered-sector firms pay the same wage to unskilled labor and since each firm sells the same product for the same price, the marginal physical productivity of unskilled labor must be equated between firms. But since the aggregate covered-sector production process is linear homogeneous, it must be that firms in the covered sector are scale replicas of each other and that with competition they produce at the minimum point on their unit cost curves where their own production functions are linear homogeneous. Equality between firms' marginal productivities of unskilled labor implies equality of factor ratios, equality of marginal productivities of skilled labor, and, therefore, efficient distributions of inputs.

Notice that the question of whether a given firm will employ both mixed and pure skill composites or only one has been begged. Clearly, if only two job titles are permitted, the firm would hire skilled labor

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from only one source because of the equal-pay constraint. If more than two job titles are permitted, in a skill-bumping equilibrium, firms will be indifferent about whether they hire skilled labor from only one source and use two job titles or from both sources and use three titles.

[4]Instead of using explicit functions for demand and production, the arithmetical effects are linear approximations only and are computed from derivatives reported in the appendix for changes in wages, factor use, and product price. Social costs are computed from formulas given in the text. Let z represent an arbitrary variable whose change is to be measured. The procedure used here is to approximate $\Delta z$, the difference between the pre- and post-quota values by

$$\Delta z \cong \frac{dz}{dr}\Big|_{r=r_o} (r_1 - r_o) + \frac{dz}{dr}\Big|_{r=r_1} (r - r_1)$$

according to the assumption that changes in the range $r_o$ to $r_1$ occur without bumping, and changes in the range $r_1$ to r occur with bumping. The only concession to changing derivatives within these ranges is that if a term involving $r^2$ occurs, the geometric mean of r over the relevant range is used rather than the initial value. For example, without skill bumping,

$$\frac{db_1}{dr} = \frac{-n}{r^2} .$$

The exact change, $\Delta b_1$, for the interval $r_o$ to r is

$$\frac{\Delta b_1}{b_1^o} = \frac{-n}{r r_o} (r - r_o) .$$

[5]Because the pattern of changed effects associated with changes in parameters such as demand and substitution elasticities is fairly

obvious, I do not report calculations based on alternative assumptions. In general, the smaller the scale of the program (i.e., the smaller the quota ratio relative to the population ratio and the relative probability of being skilled), the smaller is the coverage rate, and the lower the aggregate share of covered occupations, etc., the smaller is the overall effect. Similarly, the more elastic the substitution options, either in production or consumption, the smaller are effects. By assuming high coverage rates (e.g., Program #2) and inelastic substitution, $\eta = -.5$ and $\sigma_1 = \sigma_2 = 1$, I simply overstate effects.

## IV. ONE SECTOR, CONTINUOUS SKILL

One of the commonly assumed results of quotas directed only at skilled labor is that incentives are created for long-run minority/majority convergence in skill distributions. Without skill bumping, this effect is clearly seen in the one-sector/two-skill model since for minorities, earnings of skilled labor would rise relative to earnings of unskilled labor.[1] For majority workers, average wages of skilled laborers would fall as some become unemployed, and returns to investments in skill would fall for the majority relative to the minority. This result also holds in the two-sector model without bumping, since earnings of covered-sector skilled laborers rise relative to those in the uncovered sector.[2] Of course, one anomaly of the two-sector model is that earnings of skilled labor may fall in both sectors relative to earnings of unskilled labor, so that although a quota may result in minority/majority skill convergence it may do so by lowering the average level of skill in each group. With skill bumping, effects on returns to acquiring skill are even more ambiguous.

To demonstrate this ambiguity, consider an economy-wide, one-sector case, in which skill varies continuously. Suppose further that skill is defined in terms of time units of foregone earnings, that life is infinite, and that in the pre-quota (reference) equilibrium the rate of return to acquiring skill, the fractional increment in wages associated with a unit increment in skill, is independent of skill level. The reference wage functions are

$$w_1(s) = \exp\{\beta_o + \beta_1 s\} \tag{19}$$

for the majority and

$$w_2(s) = \exp\{\alpha_o\beta_o + \alpha_1\beta_1 s\} \tag{20}$$

for the minority, with $0 < \alpha_o, \alpha_1 \leq 1$, where $1 - \alpha_o$ is the marginal discrimination coefficient against minority workers in general and

$1 - \alpha_1$ is the discrimination coefficient against minority skill. In the reference equilibrium, the rate of return to skill for majority workers is

$$\frac{d\ln w_1}{ds} = \beta_1 \tag{21}$$

and for minority workers it is

$$\frac{d\ln w_2}{ds} = \alpha_1\beta_1 \ . \tag{22}$$

Now consider a quota with skill bumping which requires that each employed minority worker of skill greater than some minimal level, $s_o$, must be assigned to and receive the same wage as a group of $1/r$ majority workers. Assignment is assumed to be socially costless, so the quota has no immediate effect on employment and therefore no immediate effect on labor productivities.

In this form, the quota is nothing more than an income transfer. Instead of workers receiving their own perceived marginal products, they receive the average perceived product of workers in their unit.

The question of the effect of such a program on incentives for acquiring skill depends exclusively on the solution to the question of how associational units are formed. For the moment, ignore this problem and assume that majority workers of skill index $s_1$ are assigned to minority workers of index $s_2$. The wage function corresponding to this assignment is

$$\bar{w}(s_1, s_2) = \frac{w_1(s_1) + rw_2(s_2)}{1 + r} \tag{23}$$

and rates of return to acquiring skill are

$$\frac{d\ln\bar{w}(s_1, s_2)}{ds_1} = \beta_1 \left[1 + \left(\alpha_1 \frac{ds_2}{ds_1} - 1\right)\phi_1\right] \tag{24}$$

for the majority and

$$\frac{d\ln\bar{w}(s_1, s_2)}{ds_2} = \alpha_1\beta_1\left[1 + \left(\alpha_1^{-1}\frac{ds_1}{ds_2} - 1\right)\phi_2\right] \qquad (25)$$

for the majority, where

$$\phi_1 = \frac{rw_2(s_2)}{w_1(s_1) + rw_2(s_2)}, \quad \phi_2 = 1 - \phi_1, \quad \text{and} \quad \phi_1, \phi_2 > 0.$$

Clearly, assignment is the central issue since if $\alpha_1(ds_2/ds_1) > 1$ the quota will increase the rate of return to acquiring skill for majority workers and, correspondingly, reduce returns for minority workers. If $\alpha_1(ds_2/ds_1) < 1$, the opposite is true, and if $\alpha_1(ds_2/ds_1) = 1$, rates of return to skill are not affected by the quota. How, then, will assignment occur?

Under the quota, the wage function can be written as an average:

$$\bar{w}(s_1, s_2) = \frac{1}{1 + r}w_1(s_1) + \frac{r}{1 + r}w_2(s_2) \qquad (23a)$$

of the product of majority workers with weight 1/1 + r and the product of minority workers with weight r/1 + r in each unit of 1 + r workers. To a majority worker, the cost of the quota is r/1 + r of his own wage, and the return is r/1 + r of the wage of the minority workers he is assigned to. Similarly, for a minority worker the cost is 1/1 + r of his wage, and the return is 1/1 + r of the wage of the majority workers he is assigned to. In each case, costs are independent and returns depend directly on assignment. Each worker, either minority and majority, would clearly prefer to be assigned to the most productive members of the other group. Since *all* minority workers prefer assignment to *the* most productive majority worker and since *all* majority workers prefer assignment to *the* most productive minority worker, then, if workers are free to form their own associational units, only the most productive members of each group will be able to select the partners they prefer.

In this case, the nature of assignment would be that the most productive majority worker would combine with the r most productive minority workers, the second most productive majority worker would be matched with the second r most productive minority workers and so forth. This creaming process could continue until all covered workers are matched.

Now suppose that $h_1(s)$ describes the majority population's frequency density of skill while $h_2(s)$ refers to the minority skill density. If majority workers of skill $s_1$ are assigned to minority workers of skill $s_2$, it must be that all majority workers with $s > s_1$ have been assigned to minority workers with $s > s_2$. If $N_1$ refers to the population size for the majority and $N_2$ refers to the minority population, the number of majority workers with $s > s_1$ is $N_1[1 - H_1(s_1)]$, and the number of minority workers with $s > s_2$ is $N_2[1 - H_2(s_2)]$, where $H_1(\cdot)$ and $H_2(\cdot)$ are the respective cumulative densities. With a quota ratio of r, it must be true that

$$r[1 - H_1(s_1)] = \pi[1 - H_2(s_2)] \ , \tag{26}$$

where $\pi = N_2/N_1$ is the population ratio.

Thus, Eq. (26) implicitly defines the function $s_2 = s_2(s_1)$, which can be used to evaluate wages and, therefore, returns to acquiring skill. Differentiation gives

$$-rh_1(s_1)ds_1 = -\pi h_2(s_2)ds_2 \ ,$$

so that

$$\frac{ds_1}{ds_2} = \frac{\pi}{r} \ \frac{h_2(s_2)}{h_1(s_1)} \ . \tag{27}$$

This, of course, does not help unless $s_1$ and $s_2$ are known and, unfortunately, for fairly simple skill densities, Eq. (26) contains no obvious closed-form solution. The situation is not completely hopeless. Suppose, for example, that both the minority and majority skill densities are normal, so that

$$s_1 \sim N(\mu_1, \sigma_1) \quad \text{and} \quad s_2 \sim N(\mu_2, \sigma_2) \ .$$

Define the normal statistics

$$t_1 = \frac{s_1 - \mu_1}{\sigma_1} \quad \text{and} \quad t_2 = \frac{s_2 - \mu_1}{\sigma_2}$$

and let $P(t_o)$ be the reverse cumulative for the probability that t exceeds $t_o$. Equation (26) in this case becomes

$$rP(t_1) = \pi P(t_2) \ , \tag{26a}$$

so that if $r = \pi$ it must be true that $t_1 = t_2$. In this case,

$$s_1 = \frac{\sigma_1}{\sigma_2} s_2 + \mu_1 - \frac{\sigma_1}{\sigma_2} \mu_2 \quad \text{and} \quad \frac{ds_1}{ds_2} = \frac{\sigma_1}{\sigma_2} \ .$$

Since the quota will only increase the return to acquiring skill for the minority (and, correspondingly, reduce it for the majority), if $\alpha_1^{-1}(ds_1/ds_2) > 1$, this condition reduces to a question of whether $(\sigma_1/\sigma_2) > \alpha_1$.

With the assumption of normal skill densities, the differential of Eq. (26a) is

$$\frac{ds_1}{ds_2} = \frac{\sigma_1}{\sigma_2} \ \frac{\pi}{r} \ \frac{p(t_2)}{p(t_1)} \ , \tag{27a}$$

where p(t) is the normal density at t. Inspection of tables for normal densities and their cumulatives under condition (26) shows that

$$\frac{\pi}{r} \ \frac{p(t_2)}{p(t_1)} \gtrless 1 \quad \text{as} \quad \frac{\pi}{r} \gtrless 1 \ ;$$

furthermore,

$$\frac{\pi}{r}\,\frac{p(t_2)}{p(t_1)}$$

converges to 1 as $t_1$ is increased so that $\pi/r$ differs greatly from unity; the approximation $(ds_1/ds_2) \cong (\sigma_1/\sigma_2)$ may not be too bad.

To put all of this in an empirical context, suppose that the skill index refers to years of schooling and that the quota is restricted to the upper tail of skill densities. Calculations by James Smith and me (1975) suggest that for fairly recent entrants into the labor market, percentage increments in weekly wages associated with additional schooling are roughly the same for black and white males. In contrast, the logarithmic wage variance is generally greater for blacks than for whites, but in the upper tail of the earnings distribution there is more relative dispersion among whites. These points suggest that in relevant ranges $\sigma_1 > \sigma_2$. But what of the population/quota ratio, $\pi/r$?

The modern thrust of affirmative action pressure seems to be that the target quota ratio is no greater than the population ratio. However, affirmative action pressure focuses largely on *remedial* action, which is concerned more with catching up through hiring new market entrants than with hiring earlier cohorts. In this vein it may not only be that with respect to new entrants the quota ratio exceeds the population ratio but the discrepancy between them may be large. If so, this effect can swamp the presumption that $\sigma_1 > \sigma_2$, and it may be that for covered skills, quotas have actually reduced rates of return to acquiring skills for minorities.

If this were true, is it necessary that quotas drive minority and majority skill densities apart? The answer depends on the reason that the skill distributions differ in the first place. If access to capital markets is important, there are clear incentives for skill convergence since the transfer component of the program will increase minority incomes. But, in any case, the ambiguity concerning effects on rates of return survives.

To summarize this section, assume that all minority workers with skill above some minimum level, $s_o$, are covered by a quoted ratio, r. Relative to the reference profile, post-quota wages for minorities would coincide with the reference profile for skill levels below $s_o$, would be discontinuous at $s_o$, and would lie uniformly above the reference profile for skills above $s_o$. The rate of return to acquiring skill would coincide with the reference rate, $\alpha_1\beta_1$ to $s_o$, would be undefined at $s_o$, and would lie either above, be equal to, or lie below the reference rate depending on whether $(ds_2/ds_1) \gtreqless \alpha_1$ for skills above $s_o$. If rates of return are predictors of intergeneration change in skill densities, these first-generation effects would be for the skill distribution to collapse on $s_o$, both from below and from above in the vicinity of $s_o$. Effects above $s_o$ are ambiguous.

For the majority, all workers with skill above some minimum level, $s'_o$, are presumed covered by the quota.[3] The quota profile would coincide with the reference profile for skill levels below $s'_o$, would be discontinuous at $s'_o$, and would lie below the reference profile for $s > s_o$. The rate of return to acquiring skill coincides with the reference rate to $s'_o$, is undefined at $s'_o$, and above $s'_o$ it lies either above or below the reference rate, respectively, as the return to minorities falls short of or exceeds their own reference rate. The majority incentive is to avoid $s'_o$, and first-generation changes should show this. As with minorities, for majority skill distributions, effects above $s'_o$ are ambiguous, but the case for first-generation skill convergence appears no stronger than the case for divergence.

> [H]ere the story ends;
> 'Tis to be wish'd it had been sooner done,
> But stories somehow lengthen when begun.
>
> Lord Byron, "Beppo"

FOOTNOTES

[1]This, of course, assumes a perfect capital market. If skill acquisition were to depend on family income as well as on returns to

investments in human capital, effects are ambiguous since average minority earnings would fall.

[2]With skill bumping, the effect on the wage of skilled minority relative to the average wage of unskilled minority is unclear because some are bumped, and it is possible that the return to investing in skills would fall for the minority relative to the majority.

[3]The tie between $s_o$ and $s_o'$ is obvious and is given by Eq. (26). It depends on the population/quota ratio, $\pi/r$, and on the stochastic dominance of the majority over the minority skill distribution. In any case, since the quota is presumed to be stated as "at least" r minority workers for each majority worker, $s_o' \geq s_o$, and similarly for all assignment pairs, $s_1 \geq s_2$.

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## APPENDIX

### QUOTA EFFECTS WITHOUT SKILL BUMPING

The equilibrium conditions are

$$\phi(p) = f(a_1, b_1) , \tag{A.1}$$

$$pf_a = g_a . \tag{A.2}$$

Output by sectors is

$$x = f(a_1, b_1) \tag{A.3}$$

for the covered sector and

$$y = g(a_2, b_2) \tag{A.4}$$

for the uncovered sector. Aggregate employment is given by

$$a_o = a_1 + a_2 , \tag{A.5}$$

$$b_o = b_1 + b_2 . \tag{A.6}$$

Wages by sector are

$$w_{a1} = pf_a , \tag{A.7}$$

$$w_{b1} = pf_b , \tag{A.8}$$

$$w_{a2} = g_a , \tag{A.9}$$

$$w_{b2} = g_b . \tag{A.10}$$

Using the standard notation, the covered sector's demand elasticity $\eta = \phi'x/p$, the elasticity of substitution between skilled and unskilled labor in the covered sector is $\sigma_1 = f_a f_b / x f_{ab}$ and in the uncovered sector is $\sigma_2 = g_a g_b / y g_{ab}$, which follows from the assumption that $f(\cdot)$ and $g(\cdot)$ are linear homogeneous. The share of skilled labor in the covered sector is $s_1 = b_1 f_b / x$ and in the uncovered sector is $s_2 = b_2 g_b / y$. Using the second-order Euler equations for linear homogeneous functions,

$$a_1 f_{aa} + b_1 f_{ab} = 0 = a_1 f_{ab} + b_1 f_{bb}$$

and

$$a_2 g_{aa} + b_2 g_{ab} = 0 = a_2 g_{ab} + b_2 g_{bb} ,$$

it follows that

$$\frac{df_a/dr}{f_a} = \frac{s_1}{\sigma_1}\left(\frac{db_1/dr}{b_1} - \frac{da_1/dr}{a_1}\right),$$

$$\frac{df_b/dr}{f_b} = \frac{1 - s_1}{\sigma_1}\left(\frac{da_1/dr}{a_1} - \frac{db_1/dr}{b_1}\right),$$

with similar relationships for changes in $g_a$ and $g_b$. Substituting these relationships into derivatives for conditions (A.1) to (A.10) and expressing results in terms of proportionate changes gives:

$$\eta \frac{dp/dr}{p} = (1 - s_1) \frac{da_1/dr}{a_1} + s_1 \frac{db_1/dr}{b_1} , \qquad \text{(A.1a)}$$

$$\frac{dp/dr}{p} + \frac{s_1}{\sigma_1}\left(\frac{db_1/dr}{b_1} - \frac{da_1/dr}{a_1}\right) = \frac{s_2}{\sigma_2}\left(\frac{db_2/dr}{b_2} - \frac{da_2/dr}{a_2}\right) , \qquad \text{(A.2a)}$$

$$\frac{dx/dr}{x} = (1 - s_1) \frac{da_1/dr}{a_1} + s_1 \frac{db_1/dr}{b_1} , \qquad \text{(A.3a)}$$

$$\frac{dy/dr}{y} = (1 - s_2)\frac{da_2/dr}{a_2} + s_2\frac{db_2/dr}{b_2}\,, \qquad \text{(A.4a)}$$

$$\frac{da_o}{dr} = \frac{da_1}{dr} + \frac{da_2}{dr} = 0\,, \qquad \text{(A.5a)}$$

$$\frac{db_o}{dr} = \frac{db_1}{dr} + \frac{db_2}{dr} = 0\,, \qquad \text{(A.6a)}$$

$$\frac{dw_{a1}/dr}{w_{a1}} = \frac{dp/dr}{p} + \frac{df_a/dr}{f_a}\,, \qquad \text{(A.7a)}$$

$$\frac{dw_{b1}/dr}{w_{b1}} = \frac{dp/dr}{p} + \frac{df_a/dr}{f_a}\,, \qquad \text{(A.8a)}$$

$$\frac{dw_{a2}/dr}{w_{a2}} = \frac{dg_a/dr}{g_a}\,, \qquad \text{(A.9a)}$$

$$\frac{dw_{b2}/dr}{w_{b2}} = \frac{dg_b/dr}{g_b}\,. \qquad \text{(A.10a)}$$

In the range $r_o$ to $r_1$, $b_1 = n(1 + r^{-1})$, and $(db_1/dr) = (-n/r^2)$ is given exogenously. With this, conditions (A.1a) and (A.2a) after substitution of (A.5a) and (A.6a) to eliminate $db_2/dr$ and $da_2/dr$ from (A.2a) can be solved for $dp/dr$ and $da_1/dr$. These solutions are

$$\frac{dp/dr}{p} = -\frac{1}{\Delta}\left\{s_2\sigma_1\left[s_1\frac{a_1}{a_2} + (1 - s_1)\frac{b_1}{b_2}\right] + s_1\sigma_2\right\}\frac{db_1/dr}{b_1}\,, \qquad \text{(A.11)}$$

$$\frac{da_1/dr}{a_1} = -\frac{1}{\Delta}\left[s_2\sigma_1\eta\left(\frac{b_1}{b_2}\right) + s_1(\sigma_1 + \eta)\sigma_2\right]\frac{db_1/dr}{b_1} \qquad \text{(A.12)}$$

with $\Delta = (1 - s_1)\sigma_1\sigma_2 - s_1\sigma_2\eta - s_2\sigma_1\eta(a_1/a_2) > 0$. These solutions in turn can be plugged into (A.3a), (A.4a), and (A.7a) to (A.10a) to compute rates of change in wages and output by sector. Since qualitative effects are of some interest, a few results are reported. First,

$$\frac{dw_a}{dr} \propto \sigma_1 \left[ s_1 \left( \frac{a_1}{a_2} \right) + (1 - s_1) \frac{b_1}{b_2} \right] - s_1\eta \left( \frac{b_1}{b_2} - \frac{a_1}{a_2} \right), \qquad \text{(A.13)}$$

$$\frac{dw_{b2}}{dr} \propto - \frac{dw_a}{dr} \qquad \text{(A.14)}$$

(a numeriare condition), and

$$\frac{dw_{b1}}{dr} \propto \frac{dw_a}{dr} + \sigma_2 - s_2\eta \left( \frac{b_1}{b_2} - \frac{a_1}{a_2} \right). \qquad \text{(A.15)}$$

The assumption that the covered sector is skill intensive, that is, $b_1/b_2 - a_1/a_2 > 0$, is sufficient for $dw_a/dr > 0$, $dw_{b2}/dr < 0$. Furthermore, effects on the covered-sector skilled/unskilled wage ratio are ambiguous:

$$\frac{d(w_{b1}/w_a)}{dr} \propto \sigma_2 + s_2\eta \left( \frac{b_1}{b_2} - \frac{a_1}{a_2} \right). \qquad \text{(A.16)}$$

This is, of course, relevant for determining effects of quotas on the rate of return to acquiring skill. If demand for covered-sector output is very elastic relative to substitution options in the uncovered sector, and if the two sectors differ greatly in skill intensity, relative returns to skill will fall for both minority and majority workers as a result of the quota.

QUOTA EFFECTS WITH SKILL BUMPING

At the point $r_1$, the condition

$$w_{b1} = w_{b2}\left(\frac{1 + r}{1 + r\Theta f_a/f_b}\right) \qquad (A.17)$$

is satisfied, and for quota ratios above $r_1$ this condition augments (A.1) and (A.2) as the model's full set of equilibrium criteria. Further, since the effective use of some unskilled labor is lost, (A.5) is replaced by

$$a = a_o - r(1 - \Theta)\tilde{b}_1 = a_1 + a_2 \ , \qquad (A.18)$$

where $\tilde{b}_1$ refers to number of skilled majority workers in mixed composites, that is, the number who work in conjunction with bumped unskilled minority workers. Evaluated at $r_1$ (where $\tilde{b}_1 = 0$),

$$\frac{da}{dr} = -r(1 - \Theta)\frac{d\tilde{b}_1}{dr} = \frac{da_1}{dr} + \frac{da_2}{dr} \ . \qquad (A.18a)$$

The linear derivatives for this system of three equilibrium equations are solved for $da_1/dr$, $dp/dr$, and $d\tilde{b}_1/dr$, and these solutions along with (A.18a) and (A.6a) can be plugged into conditions (A.3a), (A.4a), and (A.7a) to compute incremental effects on sectorial outputs and wages for the interval $r_1$ to $r$. For these calculations, it is important to note that

$$b_1 = b_1' + \tilde{b}_1 \ ,$$

where $b_1' = n(1 + r^{-1})$ is skill employment in the pure composites. Notice also that in $d\tilde{b}_1/dr$ there are two components. One is an endogenous response to $db_1'/dr$. This part of $d\tilde{b}_1/dr$ is positive and serves to offset the otherwise growing wedge between $w_{b1}$ and $w_{b2}$ generated by an increased quota. The other component of $d\tilde{b}_1/dr$ is negative and refers to growth in $(1 + r)/(1 + r\Theta f_a/f_b)$ (see condition A.17) associated with growth in the quota ratio, $r$.

Because the solutions do not simplify, a few additional terms are useful. Define

$$\alpha = \frac{n}{b_1 r_1^2} = -\frac{db_1'/dr}{b_1} ,$$

$$\omega = \frac{b_1}{b_2} - \frac{a_1}{a_2} - \beta$$

with

$$\beta = r(1 - \Theta)\frac{b_1}{a_1} ,$$

$$\gamma = \frac{1}{1 + r\Theta f_a/f_b} ,$$

$$\xi = \frac{1 - \Theta f_a/f_b}{(1 + r)(1 + r\Theta f_a/f_b)} .$$

With these, the solutions are:

$$\Delta' \frac{dp/dr}{p} = \frac{-\alpha}{\sigma_1\sigma_2}(s_1 - \gamma s_2)\beta + \frac{\xi s_1}{\sigma_1} + \frac{\xi s_2}{\sigma_2}\left[\frac{a_1}{a_2} + (1 - s_1)\omega\right] , \qquad (A.19)$$

$$\Delta' \frac{da_1/dr}{a_1} = \frac{-\eta\alpha}{\sigma_1\sigma_2}(s_1 - \gamma s_2)\beta - \frac{\alpha s_1}{\sigma_2}\beta + \xi\left(s_1 + \frac{s_1\eta}{\sigma_1} + \frac{s_2\eta}{\sigma_2}\frac{a_1}{a_2} + \omega\right) ,$$

(A.20)

$$\Delta' \frac{db/dr}{b_1} = \frac{-\eta\alpha}{\sigma_1\sigma_2}(s_1 - \gamma s_2)\left(\frac{b_1}{b_2} - \frac{a_1}{a_2}\right) + \frac{\alpha}{\sigma_1}\left[\gamma + s_1\frac{a_1}{a_2} + (1 - s_1)\frac{b_1}{b_2}\right]$$

$$+ \xi\left[\frac{\eta s_1}{\sigma_1} + \frac{\eta s_2(a_1/a_2)}{\sigma_2} - (1 - s_1)\right] , \qquad (A.21)$$

where

$$\Delta' = \frac{-\eta}{\sigma_1\sigma_2}(s_1 - \gamma s_2)\omega + \frac{\gamma}{\sigma_1} + \frac{a_1/a_2 + (1 - s_1)\omega}{\sigma_2} > 0 ,$$

and $da/dr = -\beta$ is evaluated as $r_1$.

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